Trajectory Optimization - Warm-start Trajectory Generation

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Outline

Why do we need to generate a warm-start trajectory?

Robustness issue with direct shooting methods

 Shooting method Optimize over actions only

 $\min_{u_1,u_2,\ldots,u_T} \sum c(x_1,u_1) + c(f(x_1,u_1),u_2) + c(f(f(x_1,u_1),u_2),u_3) + \ldots$

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Why do we need to generate a warm-start trajectory?

Highly constrained planning problems



Why do we need to generate a warm-start trajectory?

Discontinuity in the cost function or the dynamics

- Discontinuity in the cost function
 - dynamic obstacles, jay walkers
- Discontinuity in dynamics

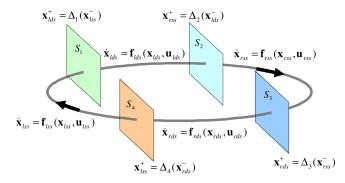
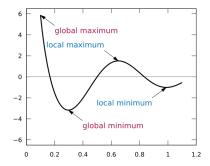


Figure: Hybrid dynamics model for 3D walking

Why do we need to generate a warm-start trajectory?

For better performance

• For better local minimum



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• For better convergence

How to generate a warm-start trajectory

Solve a simple problem first

¹Whole-body Motion Planning with Centroidal Dynamics and Full Kinematics

How to generate a warm-start trajectory

Solve a simple problem first

• Use a simple model to generate high-level trajectory first ¹ (e.g. use a lumped-mass model to optimize the CoM trajectory)

¹Whole-body Motion Planning with Centroidal Dynamics and Full Kinematics

How to generate a warm-start trajectory

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- Use a simple model to generate high-level trajectory first ¹ (e.g. use a lumped-mass model to optimize the CoM trajectory)
- Replace a difficult objective with a simple one, e.g. remove dynamic obstacles

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How to generate a warm-start trajectory

Solve a simple problem first

- Use a simple model to generate high-level trajectory first ¹ (e.g. use a lumped-mass model to optimize the CoM trajectory)
- Replace a difficult objective with a simple one, e.g. remove dynamic obstacles
- Relax constraints or remove difficult constraints
 - Relax contact [Contact-Invariant Optimization]
 - Neglect vehicle non-holonomic constraints

$$\dot{x} = \cos(\theta) v$$

$$\dot{y} = \sin(\theta)v$$

¹Whole-body Motion Planning with Centroidal Dynamics and Full Kinematics

How to generate a warm-start trajectory

Use a robust planner to generate a rough solution

²Humanoid full-body manipulation planning with multiple initial guesses and key postures, Bowei Tang, Tiuanyu Chen, Chris Atkeson

How to generate a warm-start trajectory

Use a robust planner to generate a rough solution

• Sampling-based method, such as RRT

²Humanoid full-body manipulation planning with multiple initial guesses and key postures, Bowei Tang, Tiuanyu Chen, Chris Atkeson

How to generate a warm-start trajectory

Use a robust planner to generate a rough solution

- Sampling-based method, such as RRT
- Inverse Kinematics
 - Generate a trajectory in the operational space
 - Solve IK to generate a trajectory in the robot joint space

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- Way-points
 - Key postures + interpolation ²
 - Optimize feasible states + DP + interpolation + IK 3

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- Human demonstration, e.g. using human drive input to generate warm-start trajectories

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How to generate a warm-start trajectory

Use a nominal controller

• LQR controller, e.g. one-link pendulum swing-up task

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- LQR gain scheduling controller, e.g. stopping plan
- Straight steering angle policy

How to generate a warm-start trajectory

Neighboring optimal control (NOC)

$$u = u^*(k) - K\delta x(k)$$

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How to generate a warm-start trajectory

Neighboring optimal control (NOC)

For one-link pendulum swing-up problem,

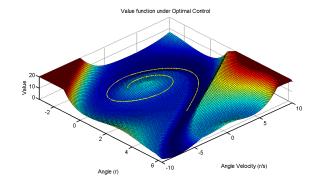


Figure: The optimal policy cost

How to generate a warm-start trajectory

Neighboring optimal control (NOC)

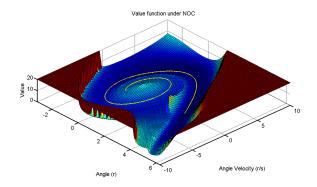


Figure: The NOC policy cost

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How to generate a warm-start trajectory

Neighboring optimal control (NOC)

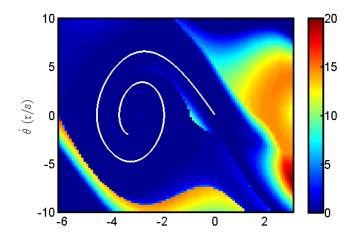
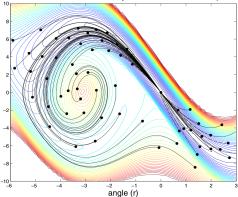


Figure: The difference between the optimal policy cost and the NOC policy cost

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How to generate a warm-start trajectory

Neighboring optimal control (NOC)



Random initial states and trajectories for one link example

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⁴Trajectory-based dynamic programming

⁵Standing balance control using a trajectory library 🗛 👍 👍 🚛 🖉 🔊

How to generate a warm-start trajectory

Collocation method

Optimize over actions and states with constraints

$$\min_{u_1, u_2, \dots, u_T, x_1, x_2, \dots, x_T} \sum_{t=1}^T c(x_t, u_t, t)$$

s.t.

$$x_t = f(x_{t-1}, u_{t-1})$$

Collocation method in general is less sensitive to the warm-start trajectory. $^{\rm 6}$

⁶Biped walking control using a trajectory library $\langle \Box \rangle \langle \Box \rangle \langle \Xi \rangle$